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A BUSY-TONE-MULTIPLE-ACCESS-TYPE SCHEME FOR PACKET-RADIO NETWORKS

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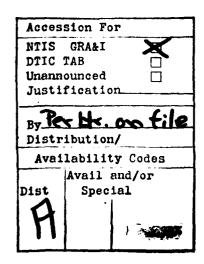
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ABSTRACT

We consider a packet radio network whose stations share a communication channel and work with an algorithm similar to the busy-tone-multiple-access protocol of ALOHA systems. In this context, two problems are treated : distributed routing and bandwidth allocation.

1. ## The Model

Consider a packet-radio network with given topology consisting of N
nodes, where data may originate at any node and is forwarded via the network,
according to some routing strategy, towards its destination. Every node has a
radio transmitter with limited range and may act as a source of data as well as
a repeater for data arriving from and destined to other nodes. Originally we
look at the situation where all nodes in the network share a common wideband
radio channel and we assume for the purpose of this paper that all transmitters
have the same transmission range, say R. The nodes are equipped with omnidirec-
tional antennas, in order to facilitate rapid and convenient deployment as well
as area coverage for mobile terminals. Consequently, two nodes i and j can
communicate directly if and only if the distance between them is R or less and
then we say that they are neighbors in the network. We denote by $N(i)$ the
collection of all neighbors of node i and by $N^2(i)$ the collection of all
neighbors of neighbors of node i, excluding node i itself and nodes that are
in $N(i)$.

Packets that originate at traffic sources have to be routed through the
network to reach their destination and since packet transmissions are received by
all neighbors, every transmitted packet should carry at each transmission the
identity of the neighbor to which it is intended. A node discards all received
packets not intended for itself.

The nature of a radio device used at each node, determines that a node may
either transmit or receive packets, but not both simultaneously. Therefore, whenever
a node i transmits a packet to node k, node k must not transmit at the same
time, and in addition, in order to avoid collisions of packets at the receiving node,
all neighbors of node k must not transmit while node i is transmitting. For
simplicity, we choose in this paper to inhibit the transmissions of all nodes in

$N(i)$ and $N^2(i)$ whenever node i transmits a packet, this guaranteeing successful transmission. This might be achieved by the following channel access scheme : each nontransmitting node continuously senses the shared channel, and whenever any activity is detected on this channel, it starts to transmit a signal on a separate, narrow band channel, called the busy-tone channel. When the activity on the shared channel ceases, the transmission on the busy-tone channel is stopped as well. It is assumed that the transmitters on both channels have the same range R.

A node is allowed to start transmission of a packet only if it detects no signal on both the shared and the busy-tone channels. Otherwise, the transmission is inhibited and the node reschedules the packet for transmission at some later time, incurring a random retransmitting delay. At this new point in time the same procedure will be invoked.

Provided that the propagation delay of the carrier is negligible, the present scheme avoids conflicts in the network. This is seen by noticing that according to this scheme, all neighbors of a transmitting node i are inhibited, since the shared channel is busy and also the neighbors of the neighbors cannot access the shared channel since all neighbors of node i transmit a signal on the busy-tone channel.

It is clear that a better scheme could be designed, in which not all neighbors and neighbors' neighbors of a transmitting node are inhibited, but only the node for which the packet is intended according to the routing procedure and its own neighbors. However, in such a scheme, the neighbors of a transmitting node will have to decode the address contained in the transmitted message, before deciding whether to transmit a signal on the busy tone channel or not. This decoding time may not be negligible, a fact that gives rise to conflicts. In this paper we restrict our attention to the channel access scheme described before.

We may also note that this scheme is a natural extension of the Busy-Tone Multiple Access (BTMA) scheme [2], that was designed for an ALOHA network, to the case of general topology radio networks.

2. Cost Function (Performance Evaluation).

   In order to evaluate the performance of a given PR network we need to
define a cost criterion. In this paper the cost function is taken to be the
average number of scheduled noncompleted transmissions, from the time a packet
enters the network until it arrives at its destination. Since every scheduled
transmission that does not take effect results in a random delay (according to
the channel access procedure described above), this average number of scheduled
transmissions is also a good indication to the average delay in the network.

   In order to express the average number of scheduled transmissions in
terms of the network parameters, we need the following simplifying assumptions
(some of which have already been mentioned) :

1) The propagation time of the carrier and the time required to detect it are
 negligible, that is zero propagation and detection time are assumed.

2) At each node in the network, the random point process defined by the points
 of time when packets are scheduled for transmission (whether they were actually
 transmitted or not) is an independent Poisson process.

3) The average time required to transmit a packet by node i is $1/\mu_i$ units of
 time (sec).

4) The shared and the busy tone channels are noise free.

5) The buffers at each node are unlimited.

6) A node cannot simultaneously transmit and receive over the shared channel.

   The critical assumptions are 1) and 2). Assumption 1), ensures, as
explained in Sec. 1, that no conflicts are possible in the network because
immediately after a node starts to transmit a packet (no two or more nodes may
start transmission simultaneously because of the Poisson assumption), all its

neighbors and the neighbors of its neighbors are inhibited. Therefore, whenever
a packet is transmitted, it is successfully received (see also Assumption 4) by
all the neighboring nodes, and in particular by the neighbor to which it is
intended. Assumption 2) is based on extensive works [7] that checked its validity
by simulation for an ALOHA network. It was shown there, that if the expectation
of the rescheduling delay is large, then Assumption 2) is a good approximation.
We still have to examine the validity of this assumption for more general topology
configurations.

Now it is relatively simple to express the average number of scheduled
transmissions of a given packet at node i, until it is actually transmitted (and
then the transmission is certainly successful). Assumption 2) implicitly says that
in steady state, the probability that a packet is actually transmitted when
scheduled, is the same whether the packet is new or has been blocked before. For
a node i, this probability is :

$$P_{s_i} = 1 - \sum_{\ell \in A(i)} S_\ell^t / \mu_\ell \qquad (1)$$

where $A(i) = i \cup N(i) \cup N^2(i)$, and S_ℓ^t is the average number of packets transmitted
by node ℓ per unit of time (sec).

Equation (1) is derived from the following simple argument : Consider a
very long interval of time T, and consider a packet that is scheduled for trans-
mission by node i in this interval. The probability that the packet is actually.
transmitted is the probability that this scheduled packet finds both the shared
and the busy tone channels idle. The portion of time that at least one of these
channels is busy during interval T is :

$$\tau = T \cdot S_i^t / \mu_i + \sum_{\ell \in N(i)} T \, S_\ell^t / \mu_\ell + \sum_{\ell \in N^2(i)} T \cdot S_\ell^t / \mu_\ell \qquad (2)$$

The first term in (2) expresses the portion of time that node i holds the channel, the second term is the portion of time that the shared channel is busy because neighbors of i are transmitting, and the third term is the portion of time that the busy tone channel is busy because neighbors of neighbors of i are transmitting.

It is clear that

$$P_{s_i} = \frac{T - \tau}{T} \tag{3}$$

and hence (1). Obviously, the condition for steady state is that $P_{s_i} > 0$ for all i.

Before proceeding, notice that in steady state, S_ℓ^t is also the average rate at which packets that are not destined to node ℓ, enter it, and is the sum of the average rate of new packets entering node ℓ (from outside of the network) denoted by S_ℓ^n, and the average rate of packets entering node ℓ from its neighbors (with destination other than ℓ). The average throughput of the network is therefore :

$$S = \sum_{\ell} S_\ell^n \tag{4}$$

where the sum is taken over all nodes in the network. S_ℓ^t is calculated by using the law of flow conservation in the network, according to the particular routing scheme used in the network. The average number of scheduled noncompleted transmissions of a given packet at node i, is simply given by

$$D_i = 1/P_{s_i} - 1 \ . \tag{5}$$

and averaged over the entire network becomes

$$D = \frac{1}{S} \sum_{i} S_i^t D_i \tag{6}$$

where the sum is taken over all nodes in the network.

The Routing Problem

Generally, the routing problem in PR networks can be specified as follows: Given the network topology and the channel access procedure at each node, determine the routing at each node such that network performance is optimized. Determination of routing in PR networks, means that whenever a node i decides to route a packet to its neighbor k, it attaches the identity of node k to the packet. All neighbors of i will receive this transmitted packet, but all, except for neighbor k, ignore it.

To specify the routing variables the following notations are used :

$\phi_{ik}(j)$ -   routing variable, expresses the fraction of flow at node i destined to node j and relayed to neighbor k. By definition $\phi_{ik}(j) = 0$ for each node k that is not a neighbor of node i, and also for $i = j$.

$S_i^n(j)$ -   input flow, expresses the rate at which packets with destination j enter node i.

$S_i^t(j)$ -   total flow, expresses the total rate at which packets with destination j transverse node i.

Clearly, the following relations hold for any node i in the network :

$$S_i^t = \sum_{j \neq i} S_i^t(j) \tag{7}$$

$$S_i^t(j) = S_i^n(j) + \sum_m S_m^t(j)\, \phi_{mi}(j) \tag{8}$$

Equation (8) expresses the law of conservation of flow at node i.

With the above notations the routing problem can be formulated as follows :

Given : Topology, channel access scheme, $\{S_i^n(j)\}$;

Minimize : Cost function $D(S_1^t, S_2^t, \ldots, S_N^t)$;

Over : $\{\phi_{ik}(j)\}$;

Constrained to :

$$\phi_{ik}(j) \geq 0 \qquad\qquad \forall i,k,j \qquad ;$$

$$\sum_{k} \phi_{ik} = 1 \qquad\qquad \forall i,j \qquad ;$$

$$S_i^t(j) = S_i^n(j) + \sum_{\ell} S_\ell^t(j)\phi_{\ell i}(j) \qquad \forall i,j \qquad ;$$

$$S_i^t = \sum_{j \neq i} S_i^t(j) \qquad\qquad \forall i \qquad ;$$

The constraint $P_{s_i} > 0$ $\forall i$, which is the condition for steady state is ignored, since it is handled implicitly by the fact that $D \to \infty$ whenever $P_{s_i} \to 0$. We are interested in a quasi-static routing algorithm that is applied distibutively [3] within the network. Actually we shall see that under some conditions, a distributed algorithm similar to those presented in [4,5,6] might be used to solve the routing problem presented above, so that the cost will be locally minimized. To show this, the following definition and two theorems are needed :

Definition : A set of routing variables ϕ is a set of non-negative numbers $\{\phi_{ik}(j)\}$, $1 \leq i,k,j \leq N$ such that

i) $\phi_{ik}(j) = 0$ $\forall i \neq j$ and $\forall k \not\in N(i)$;

ii) $\sum_{k} \phi_{ik}(j) = 1$;

iii) $\forall i,j$ $(i \neq j)$ there exists a route from i to j. In other words, there exists a set of nodes $i,k,\ell,\dots,m,j$ such that $\phi_{ik}(j) > 0$, $\phi_{k\ell}(j) > 0,\dots,\phi_{mj}(j) > 0$.

Theorem 1 : Let a set of input rates $\{S_i^n(j)\}$ and a set of routing variables ϕ be given. If the functions $\partial D/\partial S_i^t$ $\forall i$ are continuous then the set of equations (9) has a unique solution for $\partial D/\partial S_i^n(j)$.

$$
\begin{cases}
\dfrac{\partial D}{\partial S_i^n(j)} = \dfrac{\partial D}{\partial S_i^t} + \sum_k \phi_{ik}(j)\, \dfrac{\partial D}{\partial S_k^n(j)} & \forall\ i \neq j \\[4mm]
\dfrac{\partial D}{\partial S_j^n(j)} = 0
\end{cases}
\tag{9}
$$

__Theorem 2__ :   If the functions $\{\partial D/\partial S_i^t\}$ are continuous, then a sufficient condition that a set of routing variables ϕ will locally minimize D is that for all $i \neq j$ and $k \in N(i)$ (10) will hold :

$$
\frac{\partial D}{\partial S_i^t} + \frac{\partial D}{\partial S_k^n(j)} \geq \frac{\partial D}{\partial S_i^n(j)}
\tag{10}
$$

The proofs of the two theorems appear in the Appendix.   Notice that condition (10) is equivalent to

$$
\frac{\partial D}{\partial S_k^n(j)} \geq \min_{\ell:\ell \in N(i)} \left\{ \frac{\partial D}{\partial S_\ell^n(j)} \right\}
\qquad \forall\ i \neq j \text{ and } k \in N(i)
\tag{11}
$$

with equality for $\phi_{ik}(j) > 0$. (To see this multiply (10) by $\phi_{ik}(j)$, sum over k and use (9)).

From (11) it is easy to see that it is possible to develop a loop-free distributed routing algorithm similar to the algorithms that are presented in [4,5]. In principle at each iteration of the algorithm, each node i in the network decreases routing variables $\phi_{ik}(j)$ for which $\partial D/\partial S_k^n(j)$ is small. Each iteration of the algorithm will be divided in two stages : (i) the update stage at which each node i will receive $\partial D/\partial S_k^n(j)$ from its neighbors with $\phi_{ik}(j) > 0$ and will calculate $\partial D/\partial S_i^n(j)$ via (9); (ii) the rerouting stage at which the routing variables are modified according to the principle described above. If the cost function $D(S_1^t, S_2^t, \ldots, S_N^t)$ is convex, then such an algorithm leads to the global minimum cost. Unfortunately, the cost function obtained in

Sec. 2 is not convex in general, so that such an algorithm will lead only to a
local minimum.

Observe that each iteration of the algorithm requires transmission by
each node of one control message per destination to each of its neighbors [5].
The scheme for sending these control messages over radio channels is a question
for further research.

Bandwidth Allocation

In the previous sections we assumed that the shared channel is common to all nodes in the network, so that each node uses the entire bandwidth of the channel at each transmission. In this section, the following problem is addressed: For a PR network with a given total available bandwidth, can one improve performance by dividing this bandwidth? If the total bandwidth is divided into L distinct channels (L = 1 corresponds to the situation considered in previous sections), each given node will transmit over one and only one of the L distinct channels. However, in order to maintain the same neighborhood relations between nodes and the same connectivity degree in the network, it is required that each node will have L distinct receivers. With this model, there will be L sets of nodes in the network, each of them shares its common channel that is not interferring with any other channel. In order to avoid conflicts in this model, the channel access scheme described in Sec. 1 is applied in each of the L distinct channels. In addition a node that senses activity on the subchannel ℓ (where $1 \leq \ell \leq L$), transmits a signal over a corresponding busy tone channel, so that all its neighbors that use ℓ for transmission, except the transmitting node, will be silent for the period of transmission.

From the above description, it is clear that the probability of completed transmission at a node i that uses the subchannel ℓ for transmission is given here by :

$$P_{s_i} = 1 - \sum_{m \in B(i)} \frac{S_m^t}{\mu_m \cdot \delta_\ell} \qquad (12)$$

where $B(i)$ is the collection of all nodes in $A(i)$ that use the subchannel ℓ for transmission and δ_ℓ is the portion from the total bandwidth allocated to subchannel ℓ. From (5), (6) and (12) we get that the average number of scheduled transmissions of a packet in the network is :

$$D = \frac{1}{S} \left[\sum_{\ell=1}^{L} \sum_{i \in N_\ell} \frac{S_i^t}{1 - \sum_{m \in B(i)} \frac{S_m^t}{\mu_m \delta_\ell}} - \sum_{i=1}^{N} S_i^t \right] \qquad (13)$$

where N_ℓ is the set of all nodes that use the subchannel ℓ for transmission. Since the term $\frac{1}{S} \sum_{i=1}^{N} S_i^t$ depends on the routing policy and not on the bandwidth management, and since S is a constant, the cost function used in this section is reduced to :

$$D = \sum_{\ell=1}^{L} \sum_{i \in N_\ell} \frac{S_i^t}{1 - \sum_{m \in B(i)} \frac{S_m^t}{\mu_m \delta_\ell}} \qquad (14)$$

Determining L, N_ℓ and δ_ℓ, $1 \le \ell \le L$, so that the cost function D will be minimized is a very complicated problem. In this section we present two simple results : (i) in a completely connected symmetric network (i.e. each node is in the transmission range of each of the other nodes) splitting of the main channel does not improve performance; (ii) an example in which splitting the main channel does improve the performance.

Fully Connected Symmetric Network

Consider a network where all nodes are neighbors of each other. For simplicity assume that $\frac{1}{\mu_i} = 1$ and $S_i^t = S$ for each node i in the network. Assume also that the main channel is split into L separate channels. Then the cost becomes from (14) :

$$D = \sum_{i=1}^{L} \frac{S|N_i|}{1 - \frac{|N_i|S}{\delta_i}} \qquad (15)$$

where $|N_i|$ is the number of nodes in the set N_i of all nodes that share the i'th channel. Clearly $\sum_{i=1}^{L} |N_i| = N$ and $\sum_{i=1}^{L} \delta_i = 1$. When minimizing D with the constraint $\sum \delta_i = 1$, one finds (by using the Lagrange multipliers technique) that for any L and any partitioning of the nodes, the δ_i should be chosen as follows

$$\delta_i = \frac{|N_i|}{N} \qquad (16)$$

and therefore the cost becomes

$$D_{min} = \frac{NS}{1-NS} \qquad S < \frac{1}{N} \qquad (17)$$

which is the same cost as in the case when the main channel is not split at all. Therefore no improvement is noticed in the network performance by splitting the channel in this case.

Example of Performance Improvement

Consider a ciclic network with N nodes and $\frac{1}{\mu_i} = 1$ and $S_i^t = S$ $\forall$ i. When the main channel is not split, the cost is (for $N \geq 5$)

$$D = \frac{NS}{1-5S} \qquad \text{for} \quad s < \frac{1}{5} \qquad\qquad (18)$$

because $|A(i)| = 5$ $\forall$ i.

Assume now that the main channel is split in $L = 3$ equal portions, i.e. $\delta_1 = \delta_2 = \delta_3 = \frac{1}{3}$. Let the number of nodes in the network be a multiple of 3 (i.e. $N = 3k$ where k is an integer). Assume that nodes $1,4,7,\ldots,3k - 2$ use the first portion of the channel, nodes $2,5,8,\ldots,3k - 1$ use the second part and nodes $3,6,9,\ldots,3k$ use the third part. Then the cost becomes

$$D = \frac{NS}{1-3S} \qquad \text{for} \quad s < \frac{1}{3} \qquad\qquad (19)$$

since $|B(i)| = 3$ $i = 1,2,3$, showing an improvement in the network performance.

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APPENDIX

Proof of Theorem 1 :

Without loss of generality, let $j = N$ and delete the parameter j in (9). Let $\underline{F} = (F_1, F_2, \ldots, F_{N-1})^T$ and $\underline{G} = (G_1, G_2, \ldots, G_{N-1})^T$ where

$F_i = \dfrac{\partial D}{\partial S_i^t}$ and $G_i = \dfrac{\partial D}{\partial S_i^n}$ for $1 \leq i \leq N-1$. With these notations we can write (9)

as follows :

$$\underline{G} = \underline{F} + \Phi \underline{G} \qquad (A1)$$

where Φ is a $(N-1) \times (N-1)$ matrix with terms $\phi_{i\ell}$ $1 \leq i, \ell \leq N-1$.

From (A1) we have

$$G_i = \sum_{\ell} (I - \Phi)^{-1}_{i\ell} \cdot F_\ell \qquad (A2)$$

In [4, eq. A5] it is proven that the term i, ℓ of the matrix $(I-\Phi)^{-1}$ equals

$\dfrac{\partial S_\ell^t}{\partial S_i^n}$. Therefore the unique solution of (9) is

$$\frac{\partial D}{\partial S_i^n} = \sum_{\ell} \frac{\partial S_\ell^t}{\partial S_i^n} \cdot \frac{\partial D}{\partial S_\ell^t} \qquad (A3)$$

Q.E.D.

Proof of Theorem 2

Let ϕ and $\bar{\phi}$ be two sets of routing variables with corresponding flows $S_i^t(j)$, S_i^t and $\bar{S}_i^t(j)$, $\bar{S}_i^t$ respectively. Assume that ϕ satisfies (10) and that for all i $|S_i^t - \bar{S}_i^t| < \delta$ for $\delta > 0$. Then we have to show that $D(\bar{\phi}) \geqslant D(\phi)$. Let δ be chosen so that the function D is convex in the domain $|S_i^t - \bar{S}_i^t| < \delta$ for all i, and define

$$S_i^t(\lambda) = (1-\lambda)S_i^t + \lambda\bar{S}_i^t \quad \forall i \qquad 0 \leqslant \lambda \leqslant 1 \tag{A4}$$

Therefore D is a convex function of λ in this domain so that

$$- \left.\frac{dD(\lambda)}{d\lambda}\right|_{\lambda=0} \leqslant D(\bar{\phi}) - D(\phi) \tag{A5}$$

and it suffices to show that

$$\left.\frac{dD(\lambda)}{d\lambda}\right|_{\lambda=0} \geqslant 0 \quad . \tag{A6}$$

From (A4) we get that

$$\left.\frac{dD(\lambda)}{d\lambda}\right|_{\lambda=0} = \sum_i \frac{\partial D}{\partial S_i^t} (\bar{S}_i^t - S_i^t) \tag{A7}$$

so that we have to show that

$$\sum_i \frac{\partial D}{\partial S_i^t} \bar{S}_i^t \geqslant \sum_i \frac{\partial D}{\partial S_i^t} S_i^t \tag{A8}$$

To do this, multiply (10) by $\bar{\phi}_{ik}(j)$ and sum over k to get :

$$\frac{\partial D}{\partial S_i^t} + \sum_k \frac{\partial D}{\partial S_k^n(j)} \bar{\phi}_{ik}(j) \geqslant \frac{\partial D}{\partial S_i^n(j)} \tag{A9}$$

Multiplying (A9) by $S_i^t(j)$, summing first over all $j \neq i$ and then over i we obtain :

$$\sum_i \frac{\partial D}{\partial S_i^t} \bar{S}_i^t + \sum_{j \neq i,k} \frac{\partial D}{\partial S_k^n(j)} \sum_i \bar{\phi}_{ik}(j) \bar{S}_i^t(j) \geq \sum_{i,j \neq i} \frac{\partial D}{\partial S_i^n(j)} \bar{S}_i^t(j) \qquad (A10)$$

From (8) we have that

$$\sum_i \bar{\phi}_{ik}(j) \bar{S}_i^t(j) = \bar{S}_k^t(j) - S_k^n(j) \qquad (A11)$$

Substituting (A11) in (A10) yields :

$$\sum_i \frac{\partial D}{\partial S_i^t} \bar{S}_i^t \geq \sum_{j,k} \frac{\partial D}{\partial S_k^n(j)} S_k^n(j) \qquad (A12)$$

The only inequality used above was (A9) and if we substitute ϕ instead of $\bar{\phi}$ in (A9) it becomes an equality (because of (9)), so that

$$\sum_i \frac{\partial D}{\partial S_i^t} S_i^t = \sum_{j,k} \frac{\partial D}{\partial S_k^n(j)} S_k^n(j) \qquad (A13)$$

Now (A13) and (A12) yield (A8).

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